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ON THE CONDUCTIVITY OF PLASMA IN STRONG ELECTRIC AND MAGNETIC FIELDS

by

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[USSR]

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ON THE CONDUCTIVITY OF PLASMA IN STRONG ELECTRIC AND MAGNETIC FIELDS *

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by V. P. Silin

SUMMARY

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It is shown that the conductivity of plasma in strong electric and magnetic fields depends nonlinearly on the electric field strength. When the nonlinearity is strong, the cause of onset of the corresponding dependence consists in that the field strength determines the motion velocity of colliding particles. When the nonlinearity is feeble, the dependence arises when such a field drives the colliding particles out of the region of collision.

Author

* * *

Let us consider the question of plasma conductivity, when such a plasma is situated in a constant and uniform magnetic field **B**, under conditions, whereby a strong high frequency wave of circular polarization propagates in the plasma along the magnetic field. With the help of the kinetic equation for a plasma in a strong field (see [1, 2]) we shall then obtain the generalized Ohm law:

$$\frac{\partial \mathbf{j}_a}{\partial t} + [\vec{\Omega}_a \mathbf{j}_a] = \frac{e_a^2 N_a}{m_a} [\mathbf{E}_1 \cos(\omega t + \delta) - \mathbf{E}_2 \sin(\omega t + \delta)] -$$
$$- \mathbf{E}_1 [s_{1a} \sin(\omega t + \delta) + s_{2a} \cos(\omega t + \delta)] -$$
$$- \mathbf{E}_2 [s_{1a} \cos(\omega t + \delta) - s_{2a} \sin(\omega t + \delta)], \qquad (1)$$

* O PROVODIMOSTI PLASMY V SIL'NYKH ELEKTRICHESKOM I MAGNITNOM POLYAKH

where e_a is the charge; m_a is the mass; N_a is the number of particles; $\vec{\Omega}_a = e_a \mathbf{B} / m_a c$ is the gyroscopic frequency vector; j_a is the current density of type-a particles; E_1 and E_2 are equal and mutually perpendicular components of the electric field $(E_1 = E_2 = E_0)$, finally,

$$
\begin{aligned}
s_{1a} = {} & \frac{e_a^3 N_a}{m_a} \sum_b \frac{8 e_b^2 N_b}{E_0} \int_0^\infty d\tau \cos\frac{\omega\tau}{2} \int_{k_{\min}}^{k_{\max}} dk\, k \int_0^{\pi/2} d\theta \sin^2\theta \times \\
& \times \left\{ \frac{1}{m_a} \left[\tau \cos^2\theta + \frac{\sin\Omega_a\tau}{\Omega_a} \sin^2\theta \right] + \frac{1}{m_b} \left[\tau \cos^2\theta + \frac{\sin\Omega_b\tau}{\Omega_b} \sin^2\theta \right] \right\} \times \\
& \times \exp\left\{ -\frac{1}{2} v_{Ta}^2 k^2 \left[\tau^2 \cos^2\theta + 4 \frac{\sin^2\theta}{\Omega_a^2} \sin^2\frac{\Omega_a\tau}{2} \right] - \right. \\
& \left. - \frac{1}{2} v_{Tb}^2 k^2 \left[\tau^2 \cos^2\theta + 4 \frac{\sin^2\theta}{\Omega_b^2} \sin^2\frac{\Omega_b\tau}{2} \right] \right\} \times \\
& \times J_1 \left(2 \frac{E_0 k}{\omega} \left[\frac{e_a}{m_a(\omega - \Omega_a)} - \frac{e_b}{m_b(\omega - \Omega_b)} \right] \sin\theta \sin\frac{\omega\tau}{2} \right). \qquad (2)
\end{aligned}
$$

In formula (2) $v_{Ta} = \sqrt{\varkappa T_a / m_a}$ is the thermal velocity; J_1 is th Bessel function; k_{min} is equal by order of magnitude to the inverse Debye radius of ions, and k_{max} is determined, as usual, by the nonapplicability of the perturbation theory or of classical mechanics, with the only difference, that we should utilize for particle energy the expression taking into account the oscillation of the particle under the effect of the electric field. The corresponding expression for s_2 differs from (2) by the mere substitution of $\cos \omega\tau/2$ by sine.

When obtaining the equation (1), we neglected the spatial dispersion as this is usually done in the theory of conductivity, conditioned by by collisions of particles. We shall take interest in the case of frequencies, fairly small by comparison with the electron Langmuir frequency of the plasma, and we shall limit ourselves by the case of singly ionized ions. We then may neglect s_2 by comparison with s_1, while for s_{1e}, corresponding to electronic current, we shall have

$$
\begin{aligned}
s_{1e} = {} & \frac{e N_e}{m^2 v_{Te}^2} \frac{8 e_i^2 N_i}{E_0} \int_{\varkappa_{\min}}^{\varkappa_{\max}} \frac{d\varkappa}{\varkappa} \int_0^{\pi/2} d\theta \sin^2\theta \int_{-\infty}^0 d\xi \left[\frac{d}{d\xi} J_1(\varkappa\xi\eta\sin\theta) \right] \times \\
& \times \exp\left\{ -\varkappa^2 \left[\xi^2 \left(1 + \frac{m}{M} \right) \cos^2\theta + \sin^2\theta \left(\sin^2\xi + \frac{M}{m} \sin^2\frac{m}{M}\xi \right) \right] \right\}. \qquad (3)
\end{aligned}
$$

Here $\varkappa = k\sqrt{2}v_{Te}\Omega_e^{-1}$, $\eta = \sqrt{2}v_E / v_{Te}$, where $v_E = E_0[e/m(\omega-\Omega_e) - e_i/M(\omega-\Omega_i)]$ is the velocity of the relative motion of the electron and ion in the magnetic and electric fields. It is natural, that such an expression makes sense in conditions, when collisions constitute a small effect. For plasma conductivity across a strong magnetic field, the last condition, which is the only one of interest to us, in fact takes place. Finally, $\xi_{max}(\varkappa) = \varkappa^{-3/2}\varkappa_{max}^{1/2}$ is the maximum value, to which the integration over ξ should be carried. Such a value arises from the condition of applicability of the perturbation theory, laid at the basis of the initial kinetic equation for charged particles in strong fields [1, 2].

Within the bounds of a very strong field, when $\eta \gg 1$, we obtain form (3)

$$s_{1e} = \frac{c^3 N_e}{m^2}\frac{4\pi e_i^2 N_i}{E_0^3}\frac{\mathrm{sgn}\,[e/m(\omega-\Omega_e) - e_i/M(\omega-\Omega_i)]}{[e/m(\omega-\Omega_e) - e_i/M(\omega-\Omega_i)]^2}\ln\frac{r_{\text{экр}}}{r_{\min}}, \qquad (4)$$

where $r_{\text{экр}} = 1/k_{\min}$, $r_{\min} = 1/k_{\max}$. The dependence on the field, arising in formula (5) corresponds qualitatively to the substitution in the effective collision frequency of the thermal velocity by the velocity of oscillations. Such an effect corresponds to strong nonlinearity.

One may speak of feeble nonlinearity in conditions, when the thermal velocity of electrons is great by comparison with v_E. However, even in such conditions the presence of the electric field leads to qualitative effects, if at the same time, the magnetic field is sufficiently great. Below, we shall precisely turn to such a case, when the gyroscopic radii of particles are smaller than the shielding radius of the Coulomb field. At the same time, from formula (3) we shall have for the isothermic plasma :

$$s_{1e} = eN_e[e/m(\omega-\Omega_e) - e_i/M(\omega-\Omega_i)]\{\nu^{\Omega}_{\text{эфф}} + \delta\nu_\perp\}. \qquad (5)$$

Here

$$\nu^{(\Omega)}_{\text{эфф}} = \frac{4}{3}\frac{\sqrt{2\pi}\,e^2 e_i^2 N_i}{m^2 v_{Te}^3}\ln\frac{v_{Te}}{\Omega_e r_{\min}}, \quad \delta\nu_\perp = \frac{\sqrt{2\pi}\,e^2 e_i^2 N_i}{m^2 v_{Te}^3}L_1, \qquad (6)$$

$$L_1 = \frac{8}{\sqrt{\pi}} \int\limits_{\varkappa_{\min}}^{1} d\varkappa \int\limits_0^{\pi/2} d\theta \sin^3\theta \int\limits_0^{\xi_{\max}(\varkappa)} d\xi \exp\left\{-\varkappa^2\left[\xi^2\left(1+\frac{m}{M}\right)\cos^2\theta + \right.\right.$$

$$\left.\left. + \psi(\xi)\sin^2\theta\right]\right\} J_1'(\varkappa\xi\eta\sin\theta),\ \psi(\xi) = \sin^2\xi + (M/m)\sin^2(\xi m/M). \quad (7)$$

The integral (7) is determined by contribution of collisions with the **sighting** parameters, greater than the gyroscopic radius of electrons ρ_e. In a strong magnetic field, the egress at such collisions from the region of interaction is slowed down, which leads, as is well known [2, 3] to twice logarithmic expressions, inasmuch as the time, during which the collision takes place is significantly greater than the period of gyroscopic rotation. The presence of the electric field is one of the possible causes of particle egress from the interaction region. At the same time the time of electron's egress from the collision region is by order of magnitude equal to p / v_E, where p is the sighting parameter. This fact is reflected in formula (7) by the presence of the Bessel derivative function J_1^r.

At computations with twice the logarithmic precision one may utilize for the integral (7) the approximate formula

$$L_1 = 2\int\limits_{\varkappa_{\min}}^{1} \frac{d\varkappa}{\varkappa} \int\limits_{1/\varkappa}^{\min\{\xi_{\max}(\varkappa),\, 1/\varkappa\eta\}} \frac{d\xi}{\xi} e^{-\varkappa^2\psi(\xi)}. \quad (8)$$

A comparatively simple consideration allows the obtaining of the following asymptotic formulas :

$$L_1 = \ln(r_{\text{экр}}/\rho_e)\ln(M/m) \quad (\rho_i \gg r_{\text{экр}} \gg \rho_e \gg r_{\min}M/m;\ v_{Ti}^2 \gg v_E^2); \quad (9)$$

$$L_1 = \ln(M/m)\ln(m r_{\text{экр}}/M r_{\min}) + {}^1/_2 \ln(M r_{\min}/m\rho_e)\ln(M\rho_e/m r_{\min}) \quad (\rho_i \gg r_{\text{экр}} \gg r_{\min}M/m \gg \rho_e;\quad v_{Ti}^2 \gg v_E^2); \quad (10)$$

$$L_1 = \ln(r_{\text{экр}}/\rho_e)\ln(\sqrt{\rho_e r_{\text{экр}}}/r_{\min}) \quad (\rho_e \ll \rho_i,\ r_{\text{экр}},\ r_{\min}M/m;\ v_{Te}^2 r_{\min} \gg v_E^2 r_{\text{экр}}); \quad (11)$$

$$L_1 = {}^1/_2[\ln(M/m)]^2 + \ln(r_{\text{экр}}/\rho_i)\ln(\sqrt{\rho_i r_{\text{экр}}}/r_{\min}) \quad (r_{\text{экр}} \gg \rho_i \gg \rho_e \gg (M/m)r_{\min} \gg (v_E^2/v_{Ti}^2)r_{\text{экр}}); \quad (12)$$

$$L_1 = {}^1/_2\ln(\rho_e/r_{\min})\ln(M^2 r_{\min}/m^2\rho_e) + {}^1/_2\ln(r_{\text{экр}}/\rho_i)\ln(\rho_i r_{\text{экр}}/r^2_{\min}), \quad (r_{\text{экр}} \gg \rho_i \gg (M/m)r_{\min} \gg \rho_e,\ r_{\text{экр}}v_E^2/v_{Ti}^2); \quad (13)$$

$$L_1 = \ln (r_{\text{экр}} / \rho_e) \ln (v_{Te}^2 / v_E^2)$$

$$(\rho_i, r_{\text{экр}} \gg \rho_e;\ v_E^2 \gg v_{Ti}^2 \text{ и } v_{Te}^2 r_{\min} / \rho_e); \tag{14}$$

$$L_1 = 2 \ln (v_{Te} / v_E) \ln (v_E^2 r_{\text{экр}} / v_{Te}^2 r_{\min}) +$$
$$+ {}^1/_2 \ln (v_{Te}^2 \rho_e / v_E^2 r_{\min}) \ln (v_{Te}^2 r_{\min} / v_E^2 \rho_e)$$

$$(\rho_i \gg r_{\text{экр}} \gg \rho_e \ll r_{\min} M / m; \quad v_{Te}^2 \rho_e / r_{\min} \gg v_E^2 \gg v_{Ti}^2; \tag{15}$$

or

$$r_{\text{экр}} \gg \rho_i \gg \rho_e, \quad r_{\min} M / m; \quad v_{Ti}^2 \gg v_E^2 \gg v_{Te}^2 r_{\min} / r_{\text{экр}});$$

$$L_1 = {}^1/_2 [\ln (M / m)]^2 + \ln (v_{Te}^2 / v_E^2) \ln (v_E^2 r_{\text{экр}} / v_{Te}^2 r_{\min}) +$$
$$+ {}^1/_2 \ln (v_{Te}^2 r_{\min} / v_E^2 \rho_i) \ln (v_{Te}^2 \rho_i / v_E^2 r_{\min})$$

$$(r_{\text{экр}} \gg (v_{Te}^2 / v_E^2) r_{\min} \gg \rho_i \gg \rho_e \gg (M / m) r_{\min}); \tag{16}$$

$$L_1 = {}^1/_2 [\ln (M / m)]^2 + \ln (v_{Te}^2 / v_E^2) \ln (r_{\text{экр}} / \rho_i)$$

$$(r_{\text{экр}} \gg \rho_i \gg \rho_e \gg r_{\min} M / m; \quad v_{Ti}^2 \gg v_E^2 \gg v_{Te}^2 r_{\min} / \rho_i); \tag{17}$$

$$L_1 = 2 \ln (v_{Te} / v_E) \ln (v_E^2 r_{\text{экр}} / v_{Te}^2 r_{\min}) +$$
$$+ {}^1/_2 \ln (v_{Te}^2 r_{\min} / v_E^2 \rho_e) \ln (v_{Te}^2 \rho_i / v_E^2 r_{\min})$$

$$(r_{\text{экр}} \gg (v_{Te}^2 / v_E^2) r_{\min} \gg \rho_i \gg (M / m) r_{\min} \gg \rho_e); \tag{18}$$

$$L_1 = \ln (r_{\text{экр}} / \rho_i) \ln (v_{Te}^2 / v_E^2) + {}^1/_2 \ln (\rho_e / r_{\min}) \ln (M^2 r_{\min} / m^2 \rho_e)$$

$$(r_{\text{экр}} \gg \rho_i \gg (v_{Te}^2 / v_E^2) r_{\min}, \quad (M / m) r_{\min} \gg \rho_e). \tag{19}$$

Here $\rho_i = v_{Ti} / \Omega_i$ is the radius of gyroscopic rotation of ions.

Formulas (9) – (13) do not depend on the electric field and correspond to those obtained in the work [3] with the difference, however, that in our case the applicabilities, limited by the magnitude of electric field strength, are defined as regards their thresholds. To the contrary, the expressions (14) – (19) depend essentially on the strength of the electric field. Consequently, within the bounds of feeble nonlinearity, the collision frequency is essentially nonlinear.

A high-frequency field may lead to the effective decrease of interaction in the case, when its period is less than the characteristic time of interaction (see [4], [1], [2]). The accounting of such an effect for $\Omega_e \gg \omega$ is revealed in that at integration over ξ in the integral (8), the minimum of the three expressions $\xi_{\max}(\varkappa)$, $1/\varkappa\eta$, Ω_e/ω should be taken for the upper limit. We bring forth below a series of formulas, characterizing L_4, when the influence of the electric field is also felt*

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* Without taking into account the considered electric field, the corresponding results can be found in the work by Shister [5].

$$L_1(\omega) = {}^1/_2 \ln (\rho_e / r_{\min}) \ln (M^2 r_{\min} / m^2 \rho_e) + \ln (v_{Te}^2 / v_E^2) \ln (v_E / \omega\rho_i) + $$
$$+ \ln (\omega r_{\text{экр}} / v_E) \ln (v_{Te}^2 / \omega v_E r_{\text{экр}})$$
$$(r_{\text{экр}} \gg v_E / \omega \gg \rho_i \gg r_{\min} v_{Te}^2 / v_E^2 \gg r_{\min} M / m \gg \rho_e); \qquad (20)$$

$$L_1(\omega) = {}^1/_2 \ln (\rho_e / r_{\min}) \ln (M^2 r_{\min} / m^2 \rho_e) +$$
$$+ {}^1/_2 \ln (v_{Te}^2 r_{\min} / v_E^2 \rho_i) \ln (\rho_i v_{Te}^2 / r_{\min} v_E^2) +$$
$$+ \ln (v_{Te}^2 / v_E^2) \ln (v_E^3 / v_{Te}^2 \omega r_{\min}) + \ln (\omega r_{\text{экр}} / v_E) \ln (v_{Te}^2 / \omega v_E r_{\text{экр}})$$
$$(r_{\text{экр}} \gg v_E / \omega \gg (v_{Te}^2 / v_E^2) r_{\min} \gg \rho_i \gg (M / m) r_{\min} \gg \rho_e); \qquad (21)$$

$$L_1(\omega) = {}^1/_2 [\ln (M / m)]^2 + {}^1/_2 \ln (v_{Te}^2 r_{\min} / v_E^2 \rho_i) \ln (v_{Te}^2 \rho_i / v_E^2 r_{\min}) +$$
$$+ \ln (v_{Te}^2 / v_E^2) \ln (v_E^3 / \omega r_{\min} v_{Te}^2) + \ln (\omega r_{\text{экр}} / v_E) \ln (v_{Te}^2 / \omega v_E r_{\text{экр}})$$
$$(r_{\text{экр}} \gg v_E / \omega \gg (v_{Te}^2 / v_E^2) r_{\min} \gg \rho_i \gg \rho_e \gg (M / m) r_{\min}); \qquad (22)$$

$$L_1(\omega) + {}^1/_2 [\ln (M / m)]^2 + \ln (v_{Te}^2 / v_E^2) \ln (v_E / \omega\rho_i) +$$
$$+ \ln (\omega r_{\text{экр}} / v_E) \ln (v_{Te}^2 / \omega v_E r_{\text{экр}}) \qquad (23)$$
$$(r_{\text{экр}} \gg v_E / \omega \gg \rho_i \gg \rho_e \gg r_{\min} M / m; \quad v_{Ti}^2 \gg v_E^2 \gg v_{Te}^2 r_{\min} / \rho_i);$$

$$L_1(\omega) = \ln (\omega r_{\text{экр}} / v_E) \ln (v_{Te}^2 / \omega v_E r_{\text{экр}}) + \ln (v_{Te}^2 / v_E^2) \ln (v_E^3 / \omega r_{\min} v_{Te}^2) +$$
$$+ {}^1/_2 \ln (v_{Te}^2 r_{\min} / v_E^2 \rho_e) \ln (v_{Te}^2 \rho_e / v_E^2 r_{\min}),$$
$$(\rho_i \gg r_{\text{экр}} \gg v_E / \omega \gg (M / m) r_{\min} \gg \rho_e;$$
$$(\Omega_e / \omega) v_E^2 \gg v_{Te}^2 r_{\min} / \rho_e \gg v_E^2 \gg v_{Ti}^2,$$

or

$$r_{\text{экр}} \; \& \; r_{\min} M / m \gg \rho_i \gg \rho_e; \quad r_{\text{экр}} \gg v_E / \omega \gg (v_{Te}^2 / v_E^2) r_{\min}; \qquad (24)$$
$$v_{Ti}^2 \gg v_E^2 \gg (r_{\min} / r_{\text{экр}}) v_{Te}^2);$$

$$L_1(\omega) = \ln (v_{Te}^2 / v_E^2) \ln (v_E / \omega\rho_e) + \ln (\omega r_{\text{экр}} / v_E) \ln (v_{Te}^2 / \omega v_E r_{\text{экр}}) \qquad (25)$$
$$(\rho_i, \; r_{\text{экр}} \gg \rho_e \gg r_{\min} M / m; \; v_E^2 \gg v_{Ti}^2 \text{ и } v_{Te}^2 r_{\min} / \rho_e; \; r_{\text{экр}} \gg v_E / \omega \gg \rho_e);$$

$$L_1(\omega) = \ln (v_{Te} / v_E) \ln (v_E \Omega^2 / v_{Te} \omega^2)$$
$$(r_{\text{экр}} \gg v_{Te} / \omega, \; v_E^2 \gg v_{Ti}^2 \text{ и } v_{Te}^2 r_{\min} / \rho_e); \qquad (26)$$

$$L_1(\omega) = [\ln (v_{Te} / v_E)]^2 + 2 \ln (v_{Te} / v_E) \ln (v_E^3 / v_{Te}^2 r_{\min} \omega) +$$
$$+ {}^1/_2 \ln (v_{Te}^2 r_{\min} / v_E^2 \rho_e) \ln (v_{Te}^2 \rho_e / v_E^2 r_{\min}),$$
$$(r_{\text{экр}} \gg v_{Te} / \omega \gg (v_{Te}^3 / v_E^3) r_{\min}; \quad v_{Ti}^2 \ll v_E^2 \ll v_{Te}^2 r_{\min} / \rho_e). \qquad (27)$$

It has been shown, therefore, that in strong electric and magnetic fields the conductivity depends nonlinearly on the strength of the electric field. At the same time, in conditions of strong nonlinearity, the cause of emergence of the corresponding dependence consists in that the strength of the field determines the motion velocity of colliding particles. In condition of feeble nonlinearity, the dependence on the electric field arises when such a field drives the colliding particles out of the collision region.

*** THE END ***

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